COURSE: INTRODUCTION TO FORMAL ARGUMENTATION

L2: ABSTRACT ARGUMENTATION (basic concepts)

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What’s abstract argumentation?

Abstract argumentation focuses on this aspect

5) DEFINITION OF ARGUMENT STATUS

4) DEFINITION OF ATTACK BETWEEN ARGUMENTS

3) DEFINITION OF CONFLICT BETWEEN ARGUMENTS

2) DEFINITION OF WHAT AN ARGUMENT IS

1) UNDERLYING LANGUAGE

\[ AF = \langle A, \rightarrow \rangle \]  

\( A \): set of “arguments”
- Their origin and structure is left unspecified
\( \rightarrow \): a binary relation of attack, i.e. \( \rightarrow \subseteq A \times A \)
- Its origin is left unspecified

Graphical representation as a directed graph [defeat graph]
Abstract argumentation: Dung’s model

**EXAMPLE**

Example of instantiation:

\( \beta \): it will be dry in London since there will be sunshine according to BBC

\( \gamma \): it will be wet in London since there will be rain according to CNN

\( \alpha \): according to previous experiences, BCC forecasts are unreliable
Abstract argumentation: Dung’s model

**A MORE COMPLEX EXAMPLE**

Fig. 19. The argumentation framework $AF_j$ for the Popov v. Hayashi case from [44].

[Wyner & Bench-Capon, 2007]
Argumentation framework: the task

ARGUMENT EVALUATION:

GIVEN AN ARGUMENTATION FRAMEWORK, DETERMINE THE JUSTIFICATION STATE (ALSO CALLED DEFEAT STATUS) OF ARGUMENTS, IN PARTICULAR: WHAT ARGUMENTS EMERGE UNDEFEATED FROM THE CONFLICT, I.E. ARE ACCEPTABLE?

UNDERLYING ASSUMPTION

For the sake of argument evaluation, all information from the lowest 4 layers can be represented by a directed graph
Plan of the lecture

- Abstract argumentation
  - main argumentation semantics
  - dialectical proof theories
Argumentation semantics

- Specification of a method/criteria for argument evaluation, i.e. to determine, given a set of arguments, their “defeat status”
How to identify justified arguments?

**EXTENSION** (of an argumentation framework AF)

A set of arguments that can “survive the conflict together”

**REQUIREMENTS**

- **CONFLICT-FREE:**
  
  for any $\alpha$ and $\beta$ in the set, it is not the case that $\alpha \rightarrow \beta$

**IS IT ENOUGH?**

$\alpha$ $\rightarrow$ $\beta$

\[
\begin{cases}
\text{Empty set} \\
\{\alpha\} \\
\{\beta\}
\end{cases}
\]
How to identify justified arguments?

**EXTENSION** (of an argumentation framework AF)

A set of arguments that can “survive the conflict together”

**REQUIREMENTS**

- **CONFLICT-FREE:**
  for any $\alpha$ and $\beta$ in the set, it is not the case that $\alpha \rightarrow \beta$

- **ADMISSIBLE:** conflict-free and able to defend all its elements
  for any $\alpha$ in the set, if $\beta \rightarrow \alpha$ then there is $\gamma$ in the set s.t. $\gamma \rightarrow \beta$
How to identify justified arguments?

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**IS IT ENOUGH?**

\[
\text{Empty set } \{\alpha\}
\]
How to identify justified arguments?

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**IS IT ENOUGH?**

$\alpha$ \rightarrow $\beta$ \rightarrow $\gamma$

Empty set

\{\alpha\}

\{\alpha, \gamma\}
How to identify justified arguments?

EXTENSION (of an argumentation framework AF)

A set of arguments that can “survive the conflict together”

REQUIREMENTS

- CONFLICT-FREE:
  for any $\alpha$ and $\beta$ in the set, it is not the case that $\alpha \rightarrow \beta$

- ADMISSIBLE: conflict-free and able to defend all its elements
  for any $\alpha$ in the set, if $\beta \rightarrow \alpha$ then there is $\gamma$ in the set s.t. $\gamma \rightarrow \beta$

- COMPLETE EXTENSION:
  Admissible + includes all arguments it defends
Complete-based argumentation semantics

Multiplicity of complete extensions

\[ \alpha \rightarrow \beta \rightarrow \gamma \]

\[ \alpha \rightarrow \beta \rightarrow \gamma \]

\[ \alpha \leftrightarrow \beta \]

\[ \alpha \leftrightarrow \beta \]

\[ \alpha \leftrightarrow \beta \]

\[ \alpha \leftrightarrow \beta \]
(Extension-based) argumentation semantics

Argumentation framework AF

Semantics $S$

Set of (complete) extensions $\mathcal{E}_S(AF)$
From extensions to defeat status

Set of extensions \( \mathcal{E}_s(AF) \)  

Defeat/Justification Status

A common definition

- \((Skeptically)\) justified argument: belongs to all of the extensions
- Unjustified argument: does not
  - Credulously justified: belongs to at least one extension
  - Indefensible: does not belong to any extension
Unique-status vs. multiple-status semantics

**Unique-Status Semantics**

Unique extension: empty set

\[ \alpha \text{ and } \beta \text{ directly unjustified} \]

**Multiple-Status Semantics**

\[ \Rightarrow \alpha \text{ and } \beta \text{ unjustified} \]
A first family of argumentation semantics

**Grounded semantics**

\[ \mathcal{E}_{GR}(AF) = \{GE(AF)\} \]  

The least complete extension

**Stable semantics**

\[ \mathcal{E}_{ST}(AF) \]  

Complete extensions attacking external arguments

**Preferred semantics**

\[ \mathcal{E}_{PR}(AF) \]  

Maximal complete extensions
Examples

Non admissible sets

Admissible but non maximal admissible sets

... so, what are the preferred extensions?
On the grounded semantics

• A procedural definition has been introduced in previous lecture!

• Grounded extension included in all complete extensions

• The “most skeptical” approach among complete-based semantics
Informal example

1

According to BCC SUNSHINE, so it will be dry in London

According to CNN there will be rain, so it will be wet

You have no reason to prefer either of them
Informal example

1
According to BCC SUNSHINE, so it will be dry in London
According to CNN there will be rain, so it will be wet

You have no reason to prefer either of them

2
According to BCC SUNSHINE, so it will be dry in London
According to CNN there will be rain, so it will be wet

According to previous experiences, BCC forecasts are unreliable
Informal example

1

According to BCC SUNSHINE, so it will be dry in London
According to CNN there will be rain, so it will be wet

You have no reason to prefer either of them

3

According to BCC SUNSHINE, so it will be dry in London
According to CNN there will be rain, so it will be wet

According to previous experiences, BCC forecasts are unreliable

These experiences are too old to be considered reliable
Grounded semantics: example 1

Example of instantiation:

$\beta$: it will be dry in London since there will be sunshine according to BBC

$\gamma$: it will be wet in London since there will be rain according to CNN
Example of instantiation:

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\(\beta\): it will be dry in London since there will be sunshine according to BBC

\(\gamma\): it will be wet in London since there will be rain according to CNN

\(\alpha\): according to previous experiences, BCC forecasts are unreliable
Grounded semantics: example 2

Example of instantiation:

β: it will be dry in London since there will be sunshine according to BBC

γ: it will be wet in London since there will be rain according to CNN

α: according to previous experiences, BCC forecasts are unreliable
Example of instantiation:

$\beta$ : it will be dry in London since there will be sunshine according to BBC

$\gamma$ : it will be wet in London since there will be rain according to CNN

$\alpha$ : according to previous experiences, BCC forecasts are unreliable

$\alpha'$ : these experiences are too old
Another informal example

Massimiliano is going to watch the TV this evening since there is an important football match.

Massimiliano should meet an old friend this evening, and this is important.

Massimiliano should work this evening, since the deadline of a paper is not very far away, unless there is something else really important to do.

Massimiliano usually does not work during the evening, so we may think he is not going to work this evening.

SO WHAT?
Dung representation

A = Massimiliano is going to watch the TV this evening since there is an important football match
B = Massimiliano should meet an old friend this evening
C = Massimiliano is going to work this evening, since the deadline is not very far away, unless there is something else really important to do
D = Massimiliano is not going to work this evening, since he usually does not work during the evening
Floating arguments: a problem for grounded semantics

- Sometimes a more discriminative behavior is desirable

THE CASE OF FLOATING ARGUMENTS

![Diagram showing Grounded Semantics and What we (may) want]

- The reason of this “skeptical behavior” relies on single-status approach

Let us consider multiple status approaches!
On the Stable Semantics

Stable extension = conflict-free set attacking all outside arguments

for any \( \alpha \) and \( \beta \) in the set,
it is not the case that \( \alpha \rightarrow \beta \)

THE CASE OF FLOATING ARGUMENTS

\[ \mathcal{E}_{ST}(AF) = \{ \{\alpha, \delta\}, \{\beta, \delta\} \} \quad \Rightarrow \quad \delta \text{ is justified} \]
Odd-length cycles: a problem for stable semantics

No stable extension exists!

\[ \mathcal{E}_{ST}(AF) = \emptyset \quad \text{which is different w.r.t.} \quad \mathcal{E}_S(AF) = \{\emptyset\} \]

An unsatisfactory patch

Stable extensions =
- conflict-free sets attacking all outside arguments, if there is one
- \{\emptyset\}, otherwise
Stable Semantics: an unsatisfactory patch

Stable extensions =
- conflict-free sets attacking all outside arguments, if there is one
  - \{ \emptyset \}, otherwise

\[ \mathcal{E}_{ST}(AF) = \{ \emptyset \} \implies \beta \text{ NOT justified!!!} \]
Preferred extension: maximal (under \( \subseteq \)) admissible set

\[ \mathcal{E}_{PR}(AF) = \mathcal{E}_{ST}(AF) = \{ \{\alpha, \delta\}, \{\beta, \delta\} \} \]

(\( \delta \) is justified)
Preferred semantics and odd-length cycles

- Preferred semantics handles *odd-length cycles* “better” than stable semantics (as the grounded semantics)
- Preferred semantics handles the case of floating arguments “better” than grounded semantics (as stable semantics)

\[
\mathcal{E}_{PR}(AF) = \{\{\beta\}\}\]

\[
\mathcal{E}_{ST}(AF) = \emptyset
\]

\[
\mathcal{E}_{GE}(AF) = \{\{\beta\}\}
\]
“Traditional” semantics


GROUNDDED EXTENSION: THE MOST "SKEPTICAL"!

STABLE EXTENSIONS

PREFERRED EXTENSIONS
Coincidence between semantics

**Well-founded AF**

There exists no infinite sequence $A_0, A_1, ...$ such that for each $i$, $A_{i+1}$ attacks $A_i$. (i.e. *acyclic* in the finite case)

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

**Limited controversial AF**

There exists no infinite sequence $A_0, A_1, ...$ such that for each $i$, $A_{i+1}$ is controversial wrt $A_i$ (i.e. *no odd-length cycles* in the finite case).

Every limited controversial AF is coherent, i.e. each preferred extension is stable (equivalently, preferred and stable extensions coincide).
A first family of argumentation semantics

Grounded semantics

$$\mathcal{E}_{GR}(AF) = \{GE(AF)\}$$  The least complete extension

Stable semantics

$$\mathcal{E}_{ST}(AF)$$  Complete extensions attacking external arguments

Preferred semantics

$$\mathcal{E}_{PR}(AF)$$  Maximal complete extensions

ENLARGING THE FAMILY...
Semi-stable semantics: motivation

- Stable semantics
  - clashes in some cases (odd-length cycles), however:
  - a widely applied approach (default logic, stable models of logic programming, answer set programming, etc.)
  - a very credulous approach:
    justified arguments w.r.t. stable semantics are a (sometimes strict) superset of those justified w.r.t. preferred semantics

\[ \mathcal{E}_{PR}(AF) = \{\{\alpha, \delta\}, \{\beta\}\} \quad \mathcal{E}_{ST}(AF) = \{\{\alpha, \delta\}\} \]
Informal example

J says B unrel

B says L unrel

\[ \mathcal{E}_{PR}(AF) = \{ \{ \alpha, \delta \}, \{ \beta \} \} \quad \mathcal{E}_{ST}(AF) = \{ \{ \alpha, \delta \} \} \]
Semi-stable semantics: definition

Definition

\[ E \in \mathcal{E}_{\text{SST}}(\mathcal{AF}) \text{ iff} \]

\[ E \text{ is an admissible set maximizing } (E \cup \{\alpha| E \rightarrow \alpha\}) \]

Key properties

• Guarantees existence of extensions (in the finite case):
  - a maximization requirement replaces “aggressive attack”

• Coincides with stable semantics whenever stable semantics does not clash:
  - if a stable extension \( E \) exists, then \( (E \cup \{\alpha| E \rightarrow \alpha\}) \)
    includes all arguments, therefore semistable extensions \( \equiv \) stable extensions
Semi-stable semantics: examples

Example for existence

The unique admissible set is empty
⇒ trivially maximizes (E \cup \{\alpha \mid E \rightarrow \alpha\})

Example for backward compatibility
(and difference w.r.t. preferred semantics)

\[ \mathcal{E}_{PR}(AF) = \{\{\alpha, \delta\}, \{\beta\}\} \]
\[ \mathcal{E}_{SST}(AF) = \{\{\alpha, \delta\}\} = \mathcal{E}_{ST}(AF) \]
Ideal semantics (1)

- Skeptically justified arguments wrt preferred semantics do not (always) form an admissible set, e.g.

\[
\{\delta\} \text{ is not admissible}
\]

- Definition: [Dung, Mancarella, Toni’06]

\[
\mathcal{E}_{ID}(AF) = \{\text{ID}(AF)\}
\]

where ID(AF) = maximal set E:
- E is admissible, and
- E is contained in any preferred extension
Ideal semantics (2)

A unique-status approach:
- justifying a superset of the grounded extension
- maintaining admissibility (and completeness)

Examples

$$\text{GE(AF)} = \emptyset, \quad \text{ID(AF)} = \{\delta\}$$

$$\text{GE(AF)} = \text{ID(AF)} = \emptyset$$
Informal example

(Massimiliano's example where $\delta$ is based on the default assumption that he does not work in the evening)

\[ \text{GE(AF)} = \text{ID(AF)} = \emptyset \]

(A variation of Massimiliano's example where $\delta$ is not based on a default assumption, but an equally strong counterargument that he does not work, e.g. based on the presence of little children)

\[ \text{GE(AF)} = \emptyset, \quad \text{ID(AF)} = \{ \delta \} \]
'The' family of complete-based argumentation semantics

Grounded semantics

\[ \mathcal{E}_{GR}(AF) = \{GE(AF)\} \quad \text{The least complete extension} \]

Stable semantics

\[ \mathcal{E}_{ST}(AF) \quad \text{Complete extensions attacking external arguments} \]

Preferred semantics

\[ \mathcal{E}_{PR}(AF) \quad \text{Maximal complete extensions} \]

Semi-stable semantics

\[ \mathcal{E}_{SST}(AF) \quad \text{Complete extensions maximizing the range} \]

Ideal semantics

\[ \mathcal{E}_{ID}(AF) = \{ID(AF)\} \quad \text{The maximal complete extension contained in all preferred extensions} \]

The least complete extension

Complete extensions attacking external arguments

Maximal complete extensions

Complete extensions maximizing the range

The maximal complete extension contained in all preferred extensions
Labelling vs. extension-based semantics

LABELLING-BASED SEMANTICS

- Based on the notion of *labelling*
  [assignment to each argument of a label from a predefined set]
- Specifies how to derive from an argumentation framework
  a set of labellings
- Justification of arguments derived from the set of labellings

EXTENSION-BASED SEMANTICS

- Less general (at least in theory), but more common
  kind of semantics
- Based on the notion of extension
  [set of arguments “collectively acceptable”]
Labelling-based semantics definitions (1)

- Complete labelling
  - each argument labelled IN is *legally IN* (all its attackers are OUT)
  - each argument labelled OUT is *legally OUT* (there is an attacker IN)
  - each argument labelled UNDEC is *legally UNDEC*
    (there is an attacker UNDEC and no attacker is IN)

**Example**

![Diagram showing complete labelling examples](attachment:labelling_based_semantics_diagram.png)
Labelling-based semantics definitions (2)

- Commitment relation between labellings
  \[ \mathcal{L}_1 \sqsubseteq \mathcal{L}_2 \text{ iff } in(\mathcal{L}_1) \subseteq in(\mathcal{L}_2) \wedge out(\mathcal{L}_1) \subseteq out(\mathcal{L}_2) \]
  [corresponds to inclusion of the corresponding extensions]

- Grounded labelling
  - minimal (wrt \( \sqsubseteq \)) complete labelling, i.e. minimizing IN (OUT)

- Preferred labelling
  - maximal (wrt \( \sqsubseteq \)) complete labelling, i.e. maximizing IN (OUT)

- Stable labelling
  - complete labelling without arguments labelled UNDEC

- Semistable labelling
  - complete labelling minimizing arguments labelled UNDEC

- Ideal labelling
  - maximal labelling \( \mathcal{L} : \mathcal{L} \) complete and for all preferred \( \mathcal{L}_p \) \( \mathcal{L} \sqsubseteq \mathcal{L}_p \)
Relationship between extensions and labellings

**Extensions**

- \(\alpha\) \rightarrow \(\beta\)
- \(\beta\) \rightarrow \(\alpha\)
- \(\alpha\) \rightarrow \(\alpha\)
- \(\beta\) \rightarrow \(\beta\)

**Labellings**

- IN = belongs to the extension
- OUT = attacked by the extension
- UNDEC = does not belong to + not attacked by the extension

\[
\begin{align*}
\text{LABEL} & \quad \text{OF AN} \quad \text{ARGUMENT} \\
& \quad \begin{cases}
\text{IN} = \text{belongs to the extension} \\
\text{OUT} = \text{attacked by the extension} \\
\text{UNDEC} = \text{does not belong to + not attacked by the extension}
\end{cases}
\end{align*}
\]
Plan of the lecture

- Abstract argumentation
  - main argumentation semantics
  - dialectical proof theories
Dialectical proof theories

• A positive feature of argumentation: *ability to justify* conclusions
  - by providing an argument for a conclusion
  - by countering any possible objection

• This is related to the definition of admissible set:
  - a 'coherent' point of view (for any attack there is a response)

• This suggests that a proof that an argument is justified can be *dialectical*
  - an argument is justified according to a semantics S iff it can be justified
    by a PROPONENT against possible counterarguments by an OPPONENT,
    i.e. iff PRO has a *winning strategy* to defend the argument

• The specific rules of the games depend on:
  - the semantics
  - skeptical vs credulous justification
Argument game

- Played by PRO (Proponent) and OPP (Opponent)
- Begins with PRO moving an argument $x$ to defend
- OPP and PRO moves in turn with an argument attacking one of the previous arguments played by the other player
- Whenever a player $P$ moves and the other player cannot respond, $P$ wins the game
- There are additional rules that define the legal moves, characterizing a semantics:
  - $x$ is justified (in some way) iff PRO has a “winning strategy” for $x$,
  - i.e. it is always able to win the game whatever the moves of OPP
More formally

- A (partial) 'game' corresponds to a **dispute**:
  - PRO moves an initial argument $x$ it wants to defend
  - OPP and PRO take turns by moving an argument which attacks the last move of the other player

- All possible disputes corresponding to an AF in an unconstrained setting are collected in the **dispute tree** (the 'playing field' of the game)

  **Definition 6.5.** Let $AF = \langle A, R \rangle$ be an argumentation framework, and let $a \in A$. The dispute tree induced by $a$ in $AF$ is a tree $T$ of arguments, such that $T$’s root node is $a$, and $\forall x, y \in A$: $x$ is a child of $y$ in $T$ iff $xRy$.

- (Semantics-specific) constraints enforced by a **legal move function** $\phi$:
  - given a dispute $d$, $\phi$ returns a set of arguments (that can be moved)

- Given an AF and argument $a$, a legal move function $\phi$ restricts the dispute tree to one of its sub-trees, called $\phi$ **dispute-tree induced by $a$**
Example

Fig. 6.2 i) shows an argumentation framework, and ii) shows the dispute tree induced in a. iii) shows the dispute tree induced under the assumption that OPP cannot repeat moves in the same dispute (branch of the tree)

[from Caminada & Modgil 2009]
Winning strategy

• A player wins a dispute iff it makes the last move and the other player cannot respond
• Winning strategy: representing the move PRO should play for any move of OPP (and previous moves in the current dispute) to win the dispute
• Formally, a winning strategy for a is a sub-tree of the \( \phi \) tree induced by a such that:
  - it includes a non empty finite set of disputes (branches root-leaf)
  - each dispute (branch) is finite and won by PRO (last move by PRO)
  - for each dispute \( d \), for each sub-dispute \( d' \) of \( d \) ending with \( x \) played by PRO, then for any argument \( y \) which attacks \( x \) in AF and that OPP can legally move against \( x \) according to \( \phi \), there is \( d'' \) extending \( d' \) with a move \( y_{\text{OPP}} \) (a move of OPP playing \( y \) against \( x \))
Example (continued)

Fig. 6.2 i) shows an argumentation framework, and ii) shows the dispute tree induced in a. iii) shows the dispute tree induced under the assumption that OPP cannot repeat moves in the same dispute (branch of the tree)

[from Caminada&Modgil 2009]
Argument games for the grounded semantics

- We want to identify the rules of the game (legal move function) s.t. $x \in \text{GE(AF)}$, i.e. $x$ is justified w.r.t. GROUNDED, iff PRO has a winning strategy

- Legal move function:
  - PRO cannot repeat previous moves in a dispute
  - OPP is free

- Intuition:
  - To justify $A$, for every attacker a counter-attacker different than $A$ is needed
  - This in turn must be justified in the same way
RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

Example

SUPPOSE WE WANT TO JUSTIFY $\delta$
**Grounded semantics**

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

```
Example

α → β₂ → γ₁
   ↓       ↓
    β₁      γ₂
    ↓       ↓
    β₃      γ₂
        → δ

PRO
```
Grounded semantics

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

Example

FIRST POSSIBLE MOVE FOR OPP

OPP
Example

**Grounded semantics**

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

Example

![Diagram showing PRO's possible moves](image)

(UNIQUE) EFFECTIVE MOVE FOR PRO
Grounded semantics

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

Example
Grounded semantics

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

PRO WINS
Grounded semantics

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

Example

OPP (the other possible move)
Grounded semantics

RULES OF THE GAME: PRO cannot repeat its moves (OPP can)

Example

PRO WINS
Example

Winning strategy for $\delta$
A more constrained rule

LEGAL MOVES:

- PRO cannot repeat its moves (OPP can)
- PRO cannot move y against x if x attacks y in turn
- PRO cannot move an x which
  > attacks itself
  > is attacked by previous moved arguments by PRO
  > attacks previous moved arguments by PRO
Argument games for credulous preferred

- We want to identify the rules of the game (legal move function) s.t. $x \in P$, where $P$ is a preferred extension (=admissible set!), iff PRO has a winning strategy
- Legal move function:
  - OPP cannot repeat previous moves in a dispute
  - PRO cannot move a self-attacking argument or an argument which attacks or is attacked by previously moved arguments by PRO
- Intuition:
  - an argument in an admissible set can defend itself
  - more generally, the defense of an argument can loop back to it
- THEOREM: $x$ is credulously justified according to preferred (admissible) semantics iff there is a winning strategy under the previous rules such that the set of arguments by PRO is conflict free
Fig. 6.5 i) shows an argumentation framework and ii) shows the dispute tree induced in a. iii) and iv) respectively shows the $\phi_{PC_1}$ and $\phi_{PC_2}$ dispute trees induced by a.

One may also - forbid OPP also to play arguments attacked by PRO(d) - and forbid PRO to repeat its moves
Example

Fig. 6.5  i) shows an argumentation framework and ii) shows the dispute tree induced in a. iii) and iv) respectively shows the $\phi_{PC_1}$ and $\phi_{PC_2}$ dispute trees induced by a.
Note: all previous results hold under the assumption that the AF is finite.