

# Description Logics

## Exercises 5

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### Reasoning with Ontologies

1. Consider the ontology formalized by the TBox  $\mathcal{T}$  and the ABox  $\mathcal{A}$  below.

$$\mathcal{T} := \left\{ \begin{array}{l} \text{Woman} \equiv \text{Female} \sqcap \text{Person}, \\ \text{Man} \equiv \text{Male} \sqcap \text{Person}, \\ \text{Woman} \sqsubseteq \neg \text{Man}, \\ \top \equiv (\text{Man} \sqcup \text{Woman}), \\ \text{Parent} \equiv \exists \text{hasChild}.\top, \\ \text{Grandparent} \equiv \exists \text{hasChild}.\text{Parent}, \\ \text{Father} \equiv \text{Man} \sqcap \text{Parent}, \\ \text{Mother} \equiv \text{Woman} \sqcap \text{Parent}, \\ \text{Husband} \sqsubseteq \exists \text{marriedTo}.\text{Woman}, \\ \text{Wife} \sqsubseteq \exists \text{marriedTo}.\text{Man}, \\ \text{Person} \sqsubseteq \exists \text{loves}.\top \end{array} \right\} \quad \mathcal{A} := \left\{ \begin{array}{l} \text{anne} : \text{Woman}, \text{pete} : \text{Person}, \\ \text{mary} : \text{Woman}, \text{john} : \text{Man}, \\ \text{alice} : \text{Woman}, \text{bob} : \text{Man}, \\ (\text{pete}, \text{anne}) : \text{marriedTo}, \\ (\text{anne}, \text{pete}) : \text{marriedTo}, \\ (\text{john}, \text{mary}) : \text{marriedTo}, \\ (\text{mary}, \text{john}) : \text{marriedTo}, \\ (\text{pete}, \text{john}) : \text{hasChild}, \\ (\text{anne}, \text{john}) : \text{hasChild}, \\ (\text{john}, \text{alice}) : \text{hasChild}, \\ (\text{alice}, \text{bob}) : \text{loves} \end{array} \right\}$$

For each meta-statement below, decide whether it holds or not, justifying your answers.

- $\mathcal{T} \models \text{Father} \sqsubseteq \text{Husband}$ ?
- $\mathcal{T} \models \text{Person} \equiv \top$ ?
- $\mathcal{A} \models \neg \text{Person} \sqsubseteq \perp$ ?
- $\mathcal{A} \models \text{anne} : \exists \text{hasChild}.\exists \text{hasChild}.\top$ ?
- $\mathcal{A} \models \text{anne} : \text{Grandparent}$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{anne} : \text{Grandparent}$ ?
- $\mathcal{T} \models \text{Man} \sqsubseteq \exists \text{loves}.\text{Woman}$ ?
- $\mathcal{A} \models \top \equiv (\text{Man} \sqcup \text{Woman})$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{alice} : \exists \text{loves}.\exists \text{loves}.\top$ ?
- $\mathcal{T} \models \text{alice} : \exists \text{loves}.\exists \text{loves}.\text{Man} \sqcup \text{Woman}$ ?
- $\mathcal{T} \models (\neg \text{Man} \sqcap \neg \text{Woman}) \sqsubseteq \text{Grandparent}$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{john} : \exists \text{loves}.\top$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{Mother} \sqcap \text{Father} \sqsubseteq \perp$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \top \sqsubseteq \perp$ ?
- $\mathcal{T} \models \forall \text{loves}.\perp \sqsubseteq \neg \text{Man} \sqcap \neg \text{Woman}$ ?
- $\mathcal{T} \models (\text{alice}, \text{bob}) : \text{loves}$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{pete} : \text{Male}$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{john} : \exists \text{marriedTo}.\forall \text{marriedTo}.\exists \text{loves}.\text{Woman}$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{anne} : \text{Wife}$ ?
- $\langle \mathcal{T}, \mathcal{A} \rangle \models \text{pete} : \neg \text{Person} \sqcup \exists \text{hasChild}.\text{Father}$ ?
- $\mathcal{T} \models \exists \text{hasChild}.\text{Father} \sqcup \text{Mother} \sqsubseteq \text{Grandparent}$ ?
- $\mathcal{T} \models \text{Woman} \sqcap \neg \text{Mother} \sqsubseteq \forall \text{marriedTo}.\neg \text{Father}$ ?

2. Let  $\mathcal{T}$  and  $\mathcal{A}$  be as in Exercise 1 above.

(a) (Classification) Classify the TBox  $\mathcal{T}$ , i.e., for each pair of atomic concept names  $A$  and  $B$ , decide whether  $A$  subsumes  $B$  or  $B$  subsumes  $A$  (or both) with respect to  $\mathcal{T}$  and draw the corresponding hierarchy of atomic concepts.

(b) (Concept satisfiability) Decide whether the following concepts are satisfiable or not with respect to the TBox  $\mathcal{T}$ :

- $\neg\text{Person}$
- $\text{Husband} \sqcap \neg\text{Man}$
- $\text{Grandparent} \sqcap \forall\text{hasChild}.\neg\text{Mother}$
- $\exists\text{hasChild}.\text{Father} \sqcap \text{Mother}$
- $\neg\text{Woman} \sqcup \text{Mother} \sqcup \forall\text{marriedTo}.\neg\text{Father}$
- $\text{Father} \sqcap \text{Mother}$
- $\text{Husband} \sqcap \forall\text{loves}.\perp$
- $\text{Wife} \sqcap \neg\text{Woman}$
- $\text{Wife} \sqcap \exists\text{marriedTo}.\text{Woman} \sqcap \forall\text{loves}.\text{Man}$
- $\text{Male} \sqcap \neg\text{Person}$

(c) (ABox consistency) Decide whether  $\mathcal{A}$  is consistent with respect to  $\mathcal{T}$ .

(d) (Realization) Determine the most specific concepts of which Anne and John are instances.

(e) (Retrieval) Determine the members of each of the following concepts:  $\text{Father} \sqcap \forall\text{hasChild}.\text{Woman}$ ,  $\text{Man} \sqcap \exists\text{marriedTo}.\forall\text{hasChild}.\text{Man}$ , and  $\exists\text{hasChild}.\forall\text{hasChild}.\exists\text{loves}.\text{Man}$ .