Description Logics
Exercises 3

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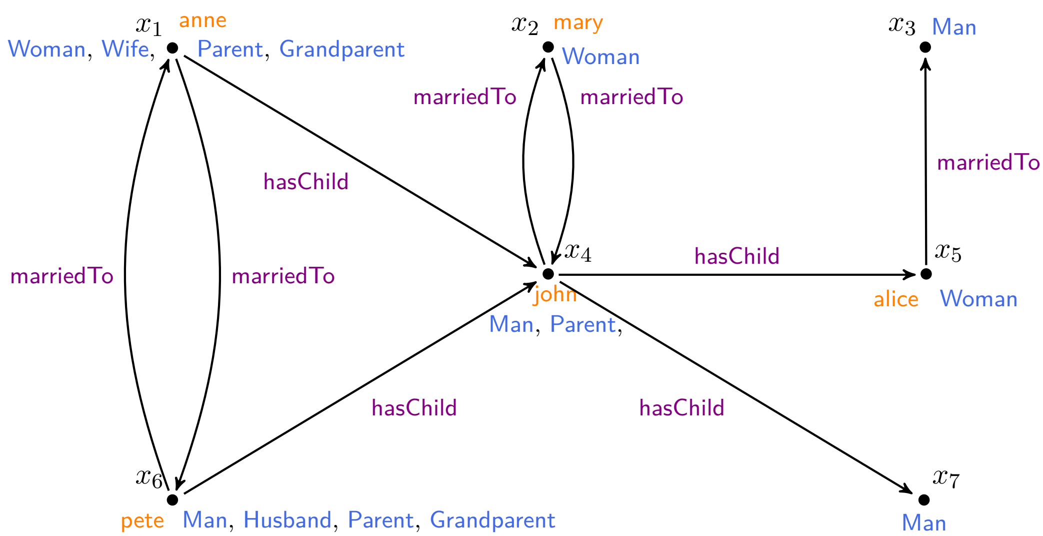
Statements

1. Let $\mathsf{N}_{\mathscr{C}} = \{\mathsf{Man}, \mathsf{Woman}, \mathsf{Husband}, \mathsf{Wife}, \mathsf{Parent}, \mathsf{Grandparent}\}$, $\mathsf{N}_{\mathscr{R}} = \{\mathsf{marriedTo}, \mathsf{hasChild}\}$ and $\mathsf{N}_{\mathscr{I}} = \{\mathsf{john}, \mathsf{mary}, \mathsf{alice}, \mathsf{pete}, \mathsf{anne}\}$. Formalize in $\mathcal{ALC}$ the following statements expressed in natural language. Whenever the natural-language description of the statement seems ambiguous, give at least one of its alternative formalizations.

1. "Women are not men".
2. "Women are not men and men are not women".
3. "Whoever has a child is a parent".
4. "Whoever marries a man is a woman".
5. "Only women marry men".
6. "Someone is either man or woman".
7. "Whoever marries a woman only has sons".
8. "Whoever has a parent child is a grandparent and only those with a parent child are grandparents". (there is no such a thing as "parent child")
9. "Wives are women married to at least one man".
10. "Husbands are men who are married only to wives".
11. "Women who are husbands are married only to women".
12. "Women married to women are wives and husbands".
13. "John is not a father or has only sons".
14. "Pete is married to someone whose children are all parents".
15. "John is married to someone whose children are all husbands".
16. "Pete is Mary's father".
17. "Anne is married to Pete".
18. "Anne has a father son".
19. "Alice is a grandchild of Pete's".
20. "Alice is a woman and a man".

Semantics of Statements

2. Let $\mathsf{N}_{\mathscr{C}} = \{\mathsf{Man}, \mathsf{Woman}, \mathsf{Husband}, \mathsf{Wife}, \mathsf{Parent}, \mathsf{Grandparent}\}$, $\mathsf{N}_{\mathscr{R}} = \{\mathsf{marriedTo}, \mathsf{hasChild}\}$ and $\mathsf{N}_{\mathscr{I}} = \{\mathsf{john}, \mathsf{mary}, \mathsf{alice}, \mathsf{pete}, \mathsf{anne}\}$. Moreovr, let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be an interpretation for $\mathsf{N}_{\mathscr{C}} \cup \mathsf{N}_{\mathscr{R}} \cup \mathsf{N}_{\mathscr{I}}$ of which the graphical representation is given by the figure below.



For each statement α formalized in Exercise 1 above, decide whether $\mathcal{I} \Vdash \alpha$ or $\mathcal{I} \nVdash \alpha$.

3. Let $\mathsf{N}_{\mathscr{C}} = \{\mathsf{Man}, \mathsf{Woman}, \mathsf{Husband}, \mathsf{Wife}, \mathsf{Parent}, \mathsf{Grandparent}, \mathsf{Father}, \mathsf{Mother}\}$, $\mathsf{N}_{\mathscr{R}} = \{\mathsf{marriedTo}, \mathsf{hasChild}\}$ and $\mathsf{N}_{\mathscr{I}} = \{\mathsf{john}, \mathsf{mary}, \mathsf{alice}, \mathsf{pete}, \mathsf{anne}\}$, and consider the knowledge base (ontology) given by the TBox $\mathcal{T}$ and the ABox $\mathcal{A}$ below. Show three interpretation satisfying this knowledge base and two interpretations not satisfying it.

$$\mathcal{T} := \left\{ \begin{array}{c} \mathsf{Woman} \sqsubseteq \neg\mathsf{Man}, \\ \mathsf{Parent} \equiv \exists\mathsf{hasChild}.\top, \\ \top \equiv (\mathsf{Man} \sqcup \mathsf{Woman}), \\ \mathsf{Grandparent} \equiv \exists\mathsf{hasChild}.\mathsf{Parent}, \\ \mathsf{Husband} \sqsubseteq \exists\mathsf{marriedTo}.\mathsf{Woman}, \\ \mathsf{Wife} \sqsubseteq \exists\mathsf{marriedTo}.\mathsf{Man}, \\ \mathsf{Father} \equiv \mathsf{Man} \sqcap \mathsf{Parent}, \\ \mathsf{Mother} \equiv \mathsf{Woman} \sqcap \mathsf{Parent} \end{array} \right\} \qquad \mathcal{A} := \left\{ \begin{array}{c} \mathsf{anne} : \mathsf{Woman},\ \ \mathsf{pete} : \mathsf{Man}, \\ \mathsf{mary} : \mathsf{Woman},\ \ \mathsf{john} : \mathsf{Man}, \\ \mathsf{alice} : \mathsf{Woman}, \\ (\mathsf{pete}, \mathsf{anne}) : \mathsf{marriedTo}, \\ (\mathsf{anne}, \mathsf{pete}) : \mathsf{marriedTo}, \\ (\mathsf{john}, \mathsf{mary}) : \mathsf{marriedTo}, \\ (\mathsf{mary}, \mathsf{john}) : \mathsf{marriedTo}, \\ (\mathsf{pete}, \mathsf{john}) : \mathsf{hasChild}, \\ (\mathsf{anne}, \mathsf{john}) : \mathsf{hasChild}, \\ (\mathsf{john}, \mathsf{alice}) : \mathsf{hasChild} \end{array} \right\}$$

4. For each of the following statements, decide which ones are valid and which are not. For each valid statement, show that every interpretation satisfies it. For each statement that is invalid, show an interpretation serving as a counter-example to it.

- $\forall r.(C \sqcap D) \equiv (\forall r.C \sqcap \forall r.D)$
- $\forall r.(C \sqcup D) \equiv (\forall r.C \sqcup \forall r.D)$
- $\exists r.(C \sqcap D) \equiv (\exists r.C \sqcap \exists r.D)$
- $\exists r.(C \sqcup D) \equiv (\exists r.C \sqcup \exists r.D)$
- $\forall r.\exists r.C \equiv \exists r.\forall r.C$
- $\exists r.\forall r.C \sqsubseteq \forall r.\exists r.C$
- $\forall r.(C \sqcup D) \sqcap \forall r.\neg C \sqsubseteq \forall r.D$
- $\exists r.(C \sqcap D) \sqsubseteq \exists r.C$
- $\forall r.\top \equiv \top$
- $\exists r.\bot \equiv \bot$
- $\bot \sqsubseteq C$
- $C \sqsubseteq \top$
- $\forall r.\bot \sqsubseteq \forall r.C$
- $\exists r.C \sqsubseteq \exists r.\top$
- $\top \sqsubseteq C \sqcup \neg C$
- $\exists r.C \sqcup \forall r.\neg C \sqsubseteq \bot$