Description Logics

Exercises 2

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Concept Language

1. Let $\mathsf{N}_{\mathscr{C}} := \{\mathsf{Man}, \mathsf{Woman}, \mathsf{Parent}, \mathsf{Grandparent}\}$ and $\mathsf{N}_{\mathscr{R}} := \{\mathsf{marriedTo}, \mathsf{hasChild}\}$. Formalize in $\mathcal{ALC}$ the following concepts expressed in natural language. Whenever the natural-language description of the concept seems ambiguous, give at least one of its alternative formalizations.

1. "Married women".
2. "Fathers married to married women".
3. "Men who are single or have unmarried daughters".
4. "Men who are single or have unmarried daughters that do not have married sons".
5. "Men who have only unmarried daughters".
6. "Mothers married to married men or single men who are not parents".
7. "Men married to a woman who has only married daughters".
8. "Men who are fathers of women who are not married but are mothers."
9. "Women who are not mothers of men who are not married but are parents or are not grandparents".
10. "Men or women who are granparents whose children are married men".
11. "Grandparents with at least one daughter and whose grandchildren are all single men".
12. "Fathers with at least one daughter whose grandchildren are married to men or single mothers".
13. "Grandparents whose chidren are not parents but are married to men or married women".
14. "Women with only daughters married to unmarried men who are fathers with no child".
15. "Fathers who have children whose daughters are married to men who have no daughter but have sons".
16. "Men married to single women and whose daughters are not mothers".
17. "Women who are not men and who are mothers of grandparents with at least one daughter that is the mother of a son".

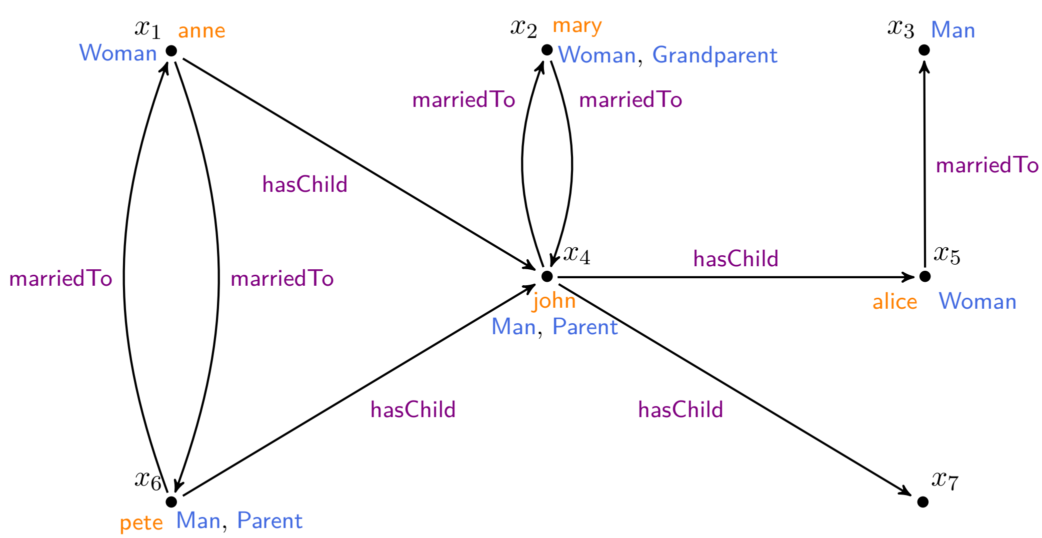
18. "Women whose children are married men and that are married to men who are not women".

19. "Women married to women who are married only to men that are women".

20. "Men who are not married to anybody and that do not have children married to women who have no child".

Semantics

2. Let $\mathsf{N}_{\mathscr{C}} := \{\mathsf{Man}, \mathsf{Woman}, \mathsf{Parent}, \mathsf{Grandparent}\}$, $\mathsf{N}_{\mathscr{R}} := \{\mathsf{marriedTo}, \mathsf{hasChild}\}$ and let $\mathsf{N}_{\mathscr{I}} := \{\mathsf{john}, \mathsf{mary}, \mathsf{alice}, \mathsf{pete}, \mathsf{anne}\}$. Moreover, let $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be an interpretation for $\mathsf{N}_{\mathscr{C}} \cup \mathsf{N}_{\mathscr{R}} \cup \mathsf{N}_{\mathscr{I}}$ of which the graphical representation is given in the figure below.



For each concept C formalized in Exercise 1 above, determine $C^{\mathcal{I}}$.

3. Let $\mathsf{N}_{\mathscr{C}} := \{\mathsf{Man}, \mathsf{Woman}, \mathsf{Parent}, \mathsf{Grandparent}\}$ and $\mathsf{N}_{\mathscr{R}} := \{\mathsf{marriedTo}, \mathsf{hasChild}\}$. Define three interpretations in which **all** the following concepts are interpreted as non-empty.

- $\mathsf{Parent} \sqcap \exists \mathsf{hasChild}.\top$
- $\mathsf{Grandparent} \sqcap \exists \mathsf{hasChild}.\mathsf{Parent}$
- $(\mathsf{Man} \sqcap \neg \mathsf{Woman}) \sqcup (\neg \mathsf{Man} \sqcap \mathsf{Woman})$
- $\exists \mathsf{hasChild}.\exists \mathsf{hasChild}.\exists \mathsf{marriedTo}.\top$
- $\exists \mathsf{marriedTo}.(\neg \mathsf{Man} \sqcap \neg \mathsf{Woman})$
- $(\mathsf{Woman} \sqcap \exists \mathsf{marriedTo}.\mathsf{Man}) \sqcup (\mathsf{Man} \sqcap \exists \mathsf{marriedTo}.\mathsf{Woman})$
- $(\mathsf{Woman} \sqcap \forall \mathsf{hasChild}.\bot) \sqcup (\mathsf{Man} \sqcap \forall \mathsf{marriedTo}.\bot)$
- $(\mathsf{Man} \sqcap \exists \mathsf{hasChild}.\top) \sqcup (\mathsf{Woman} \sqcap \forall \mathsf{marriedTo}.\bot)$
- $\exists \mathsf{hasChild}.\forall \mathsf{marriedTo}.\exists \mathsf{marriedTo}.\top$
- $\exists \mathsf{hasChild}.\exists \mathsf{hasChild}.\top \sqcap \neg \mathsf{Grandparent}$

4. For each of the following concepts, decide whether it is satisfiable or not. For each concept that is satisfiable, show an interpretation satisfying it. For each unsatisfiable concept, show that it is unsatisfiable. Below, A_1, A_2, etc. represent atomic concepts.

- $A_1 \sqcap \forall r.\neg A_1 \sqcap \exists r.\exists r.A_1$
- $\forall r.(\neg A_1 \sqcap \neg A_2) \sqcap \neg\exists r.A_1 \sqcap \neg\exists r.A_2$
- $\exists r.(A_1 \sqcup (A_2 \sqcap A_3)) \sqcap \forall r.(\neg A_1 \sqcup \neg A_3)$
- $\exists r.(A_1 \sqcup (A_2 \sqcap A_3)) \sqcap \forall r.(\neg A_1 \sqcap \neg A_3)$
- $\forall r.A_1 \sqcap \forall r.\neg A_1$
- $\forall r.\exists r.\exists r.\exists r.\exists r.\exists r.\exists r.\bot \sqcap \exists r.\forall r.\forall r.\top$
- $\forall r.A_1 \sqcap \forall r.\neg A_1 \sqcap \exists r.(A_2 \sqcup A_3)$
- $\forall r.\exists r.A_1 \sqcap \exists r.\forall r.\neg A_1$
- $\exists r.\forall r.\exists r.A_1 \sqcap \exists r.\forall r.\exists r.\neg A_1$
- $\exists r.\forall r.\forall r.A_1 \sqcap \forall r.\forall r.\exists r.A_1$